EMPLOYING WAVELET COHERENCE METHODS FOR THE MANAGEMENT AND CONSTRUCTION OF AN OPTIMAL INVESTMENT PORTFOLIO

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Abstract

In this study, the focus is on investigating comovement patterns between eight sectors of companies listed on the Pakistan Stock Exchange. This research aims to use frequency-time bands and multiscale wavelet analysis to enhance portfolio diversification and asset allocation techniques. The results indicate that comovement patterns between pairs of two sectors can be utilized for short-, medium-, and long-term investments, allowing investors to construct portfolios based on their preferred investment horizons. Additionally, the study identifies sectors that are strongly correlated, such as banks-cement, cement-auto, and chemical-auto, where investment is not recommended. The results of the study can be used by policymakers, investors, and fund managers to develop effective portfolio diversification and risk management strategies. It is expected that the findings will contribute to the literature on international finance, providing useful insights into the dynamics of comovements in the Pakistani stock market. By applying the results of this research, investors may be better equipped to navigate the complexities of the stock market, thereby increasing their chances of achieving their investment goals.

Keywords: Wavelet, Coherence; Portfolio; Diversification; Comovement

1. Introduction

Stock market comovement is a key concept in the finance literature, attracting the growing interest of investors, fund managers, academics, and regulators worldwide (Mensi et al., 2017, 2018). Comovement is an effective strategy for achieving portfolio diversification benefits in both domestic and foreign investment outlets (Dewandaru et al., 2013). Asset allocation and risk management are essential components of every investor's investment strategy. The expected benefits are realized through strong comovement of assets, which also changes the risk profile of a given portfolio. Moreover, stock markets have developed intricate global linkages and are becoming increasingly interconnected. This highlights the potential for investors to leverage the flow of money and change the allocation and duration of their investments. Technology is also a potent double-edged sword that exerts opposing forces on global financial markets (Buriev et al., 2018).

The comovement of stock prices is a widely researched phenomenon in financial economics, and it refers to the degree of correlation between the prices of different stocks. The notion of "comovement" is of particular interest to investors, portfolio managers, and policymakers who seek to understand the dynamics of the stock market and develop effective investment strategies (Al-Najjar, 2022). The literature on comovement in the stock market encompasses a wide range of empirical and theoretical studies, and the results suggest that the degree of comovement varies across stocks, markets, and time periods (T. Huang et al., 2022; Madaleno & Pinho, 2012; Mase, 2008).

Several factors drive comovement in the stock market. One of the primary drivers is the macroeconomic environment, which affects the performance of firms across different sectors and industries (Jin & Guo, 2021). For instance, an economic downturn may affect the profitability of firms, leading to a decline in their stock prices. Similarly, a positive shock to the economy, such as an increase in consumer spending or government spending, may lead to a rise in stock prices across different sectors. Another important driver of comovement is investor sentiment, which is influenced by factors such as news events, rumors, and expectations about future earnings (P H & Rishad, 2020). Positive news about one firm may spill over to other firms in the same sector or industry, leading to comovement in their stock prices.

Empirical studies have documented the existence of comovement in the stock market at various levels of aggregation (Al-Najjar, 2022; Anagnostopoulos et al., 2022; Kartal et al., 2022; Shi & Chen, 2022). At the national level, comovement is observed between the stock markets of different countries, with global economic and financial shocks affecting stock prices across borders. At the industry level, comovement is observed within sectors such as technology, healthcare, and energy, with firms in the same sector exhibiting similar patterns of stock price movements. At the

individual stock level, comovement is observed between stocks of the same company or stocks in the same industry, with common factors such as news events and earnings reports affecting their prices.

In conclusion, comovement in the stock market is a complex phenomenon that reflects the interplay of various macroeconomic, firm specific, and investor sentiment factors. While the degree of comovement varies across stocks, markets, and time periods, it is an important consideration for investors and policymakers who seek to understand the dynamics of the stock market and develop effective investment strategies. Further research is needed to explore the causes and consequences of comovement and to develop more accurate models of stock price movements.

The Pakistan Stock Exchange (PSX) is a vital platform for investors to buy and sell shares of publicly listed companies in Pakistan. The performance of the PSX is influenced by a range of economic, political, and social factors that can affect individual stocks and the market as a whole. Comovement, which refers to the tendency of two or more securities or markets to move in the same direction, is a significant feature of the PSX. Understanding the comovement of stocks in the PSX is essential for investors and policymakers, as it can help in identifying trends and opportunities for diversification.

Several studies have investigated the comovement of stocks in the PSX using different approaches and techniques. One approach is to examine the correlation between the returns of individual stocks or between the returns of the PSX index and other indices. For instance, studies have examined the comovement of the PSX index with global stock markets and found a significant positive correlation with the US, UK, and Chinese stock markets [1,2]. Another approach is to use factor models that capture the common sources of variation in stock returns. Studies have identified common factors that explain the comovement of Pakistani stocks and found that market sentiment, economic growth, and interest rates are the primary drivers of comovement [3,4].

Overall, understanding the comovement of stocks in the PSX is crucial for investors and policymakers to make informed decisions about portfolio diversification and risk management. By analyzing the comovement of stocks, investors can identify opportunities to reduce portfolio risk and enhance returns, while policymakers can use this information to design effective macroeconomic policies to promote a stable and efficient stock market. This paper reviews the literature on comovement in the PSX and discusses its implications for investors and policymakers.

The present study aims to develop an optimized portfolio by investigating the dynamic comovements among eight sectors of the stock market using a biwavelet approach at the sector level. This study benefits from new insights, particularly in analyzing time-varying features from the frequency-time domain perspective to measure the size of dynamic comovements, adding to the literature. Despite varying investment horizons, investors consider it crucial to determine the ideal

level of comovement. Ultimately, optimal comovement can lead to gains and decreasing risk patterns across various asset classes, with significant implications for risk-return strategies, investor psychology, portfolio diversification, and investor behavior (Bouranta et al., 2017; Mensi et al., 2018; Sahabuddin et al., 2022).

2. Literature Review

Comovement among stocks is a well-researched area in the finance literature, and its importance in portfolio management has been widely recognized. Comovement refers to the degree of similarity in the movement of stock prices, and it is considered an essential factor in determining the diversification benefits of a portfolio (Ang et al., 2009). Studies have shown that higher levels of comovement among stocks can lead to higher levels of systematic risk, which can significantly impact the performance of a portfolio (Zhu et al., 2018).

In recent years, the use of advanced econometric techniques has enabled researchers to better understand the dynamic comovements among stocks. A growing body of literature has emphasized the importance of considering the time-varying nature of comovements in portfolio management. For instance, studies have employed a dynamic conditional correlation model to analyze the time-varying comovements between industry sectors in the Chinese stock market (C. Huang et al., 2021). Their findings suggest that the degree of comovement varies significantly over time and across industry sectors.

Moreover, recent studies have shown that comovement among international stock markets has become increasingly important in portfolio management due to the rising globalization of financial markets (Al-Najjar, 2022). For example, studies have employed the EGARCH model to investigate the comovements between the Chinese and US stock markets (Song et al., 2022). Their results reveal that the comovements between these two markets have increased significantly over time, which has important implications for portfolio diversification.

Furthermore, studies have examined the impact of comovements on portfolio performance. For instance, (Song et al., 2022) investigated the benefits of portfolio diversification based on comovements among stocks in the Indian stock market. They found that portfolios constructed using stocks with lower levels of comovement outperformed those constructed using stocks with higher levels of comovement.

The importance of considering comovement in portfolio management has been a point of interest for researchers. Recent studies have emphasized the dynamic nature of comovements and the need to incorporate time-varying features in portfolio optimization. The increasing globalization of financial markets has also made it necessary to consider the comovements between international

stock markets in portfolio management. Future research could explore the impact of comovements on portfolio risk and return across different asset classes and geographies.

The Pakistan Stock Exchange (PSX) is the premier equity market in Pakistan, providing a platform for companies to raise capital and investors to invest in a range of securities. The PSX was formed in January 2016 after the merger of the Karachi, Lahore, and Islamabad Stock Exchanges, with the aim of providing a single platform for investors and issuers. Since then, the PSX has played a pivotal role in the economic development of the country, enabling businesses to access financing and providing investors with opportunities for wealth creation.

Research on the PSX has focused on various aspects, such as the impact of macroeconomic factors on stock prices, the efficiency of the market, and the performance of individual stocks. For instance, one study analyzed the impact of macroeconomic variables on stock prices in Pakistan using the ARDL approach (M. N. Khan & Zaman, 2012). The authors found that inflation, exchange rate, and interest rate have a significant impact on stock prices in the short and long run.

Another area of research has been the efficiency of the PSX. Studies have evaluated whether the market is weak-form efficient, semistrong efficient, or strong-form efficient. Some studies have examined the weak-form efficiency of the PSX using the random walk model (N. U. Khan & Khan, 2016; SHAMSHIR et al., 2018). The study revealed that the PSX is weakly efficient, meaning that past prices and returns do not provide any useful information for predicting future prices.

Furthermore, research has also examined the performance of individual stocks on the PSX. Some studies have analyzed the performance of the banking sector in Pakistan listed in the PSX (Rafay & Farid, 2018; Shah et al., 2022). The authors found that the banking sector had a positive impact on the overall performance of the PSX, with higher profits and returns on assets leading to higher stock prices.

In conclusion, the PSX is an important contributor to the economic development of Pakistan, and research has shown that it is influenced by various macroeconomic factors, operates efficiently, and is home to several high-performing stocks. Further research could explore the impact of regulatory policies on the PSX, the role of foreign investors, and the impact of technology on the market.

3. Materials and Methods

The data for this study were computed by combining the stock returns of publicly listed companies from eight sectors of the Karachi Stock Exchange-100 index (KSE-100). The returns were expressed as percentages by multiplying the initial differences in the daily price indices by 100. In other words, the stock returns were calculated using the following equation:

$$\boldsymbol{R_t = \log\left(P_{t/P_{t-1}}\right) \times 100 = (\log P_t - \log P_{t-1}) \times 100} \quad (1)$$

where P_t denotes index levels at time t, R_t denotes returns at time t and t is a day. Due to the unavailability of data from stock exchanges on national holidays, bank holidays, or other events, as well as time differences, it was assumed that the stock values would remain unchanged from the previous day. The first difference of each stock-price index was used to calculate the corresponding stock return, which was then reported as a percentage (multiplied by 100).

2.1 Wavelet Approach

In recent years, the wavelet technique has emerged as a popular and effective method utilized in econometric predictors in the finance literature, as well as in signal processing in engineering sectors. This method critically examines time-frequency causation between time-series economic data using signal processing methodology. Its primary goal is to establish beneficial relationships between multidimensional-based frequency brand attributes. Unlike traditional time series estimators, this method decomposes data at multiple scales to produce robust results **(Finžgar & Podržaj, 2018)**. Additionally, the movement of frequencies and scales exhibits an intriguing pattern, such as an increase in frequency (at lower scales), allowing users to differentiate between various scales and frequency ranges **(Aguiar-Conraria et al., 2008)**. This emerging method describes the econometric synergy that addresses the limitations of error terms and structural breaks, in addition to the frequency and time domains **(Aguiar-Conraria et al., 2008; Yang et al., 2017)**.

2.2 Continuous wavelet transformation (CWT)

The CWT and discrete wavelet transformation make up the bulk of the wavelet technique (DWT). However, CWT is more approachable and well liked than DWT because of its vibrant graphical presentation and straightforward interpretation **(Aguiar-Conraria & Soares, 2011)**. The DWT is a multiresolution filter-based analysis for data decomposition and recomposition, in contrast to the CWT, which is a multiresolution-based analysis for enlarging and contracting wavelet functions.

$$W_x(a,b) = \int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)dt \qquad (2)$$

The projection of the volatility and returns series by a particular wavelet ψ (.) required by the CWT. In the above equation, the scale parameter is the reflection and extension of a particular wavelet; here, the position is represented by parameter *b*, which can obtain the information of time**.**

4. Data and Preliminary Statistical Analysis

4.1 Data

This study utilized data obtained from the Pakistan Stock Exchange (www.psx.com.pk) on eight sectors: Auto, Textile, Chemicals, Cement, Oil and Gas, Pharmaceutical, Banks, and Food. The data included the closing price index of companies operating within these sectors for the period from 2010 to 2019.

4.2 Preliminary Statistical Analysis

The stock return data for firms based in Pakistan across various sectors, including auto, banks, cement, chemicals, oil & gas, pharmaceuticals, and textiles, were converted into log returns due to

their better statistical properties compared to the original returns. For the current analysis, the daily closing prices of the stocks were obtained from the Pakistan Stock Exchange. The formula used for converting the stocks was *Ln(SR)-Ln(SR+1)*.

The mean value of the stock returns was found to be nearly zero, and the null hypothesis of treating them as a normal distribution was rejected. The standard deviation of the pharmaceutical and cement sectors was found to be greater, indicating that these sectors are relatively more volatile than the other sectors.

Table 1. Descriptive Statistics

Sectors	Auto	Banks	Cement	Chemicals	Food	Oil & Gas	Pharmacuitical	Textile
Mean	0.00	0.00	0.00	0.00	0.00	3.09	0.00	0.00
Std.Dev	0.01	0.01	0.02	0.01	0.01	0.01	0.02	0.01
Skewness	0.57	0.00	-0.06	0.49	-0.06	1.03	0.08	0.71
Kurtosis	4.67	1.82	0.89	2.24	1.72	9.20	0.80	7.75

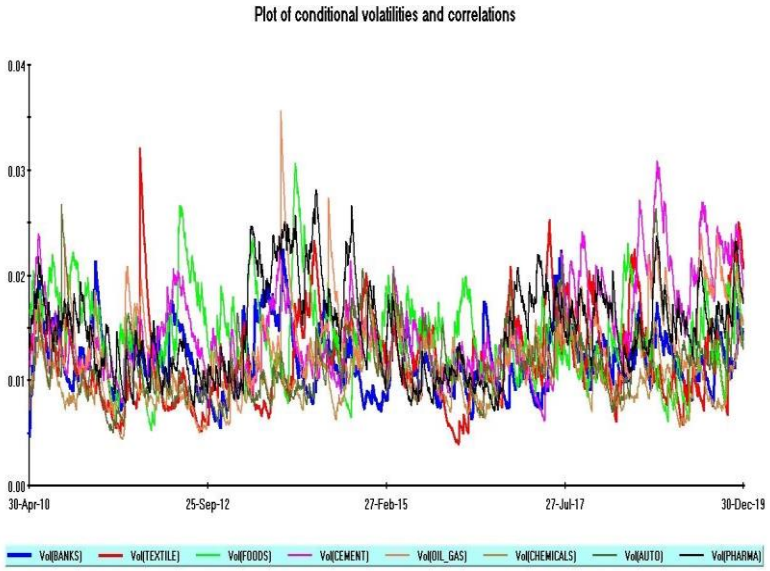


Figure 1. Conditional Volatilities

The conditional volatilities plot shows that most of the sectors move in tandem with other sectors; however, some sectors, such as Textile, Foods, Cement, Oil & Gas and Pharmaceutical, have high volatility compared with other sectors, and less volatility is observed in the Banks, Chemicals and Auto Sectors.

Table 2. Arch Effect Test

Sector	Lag	Score	C.V.	P Value	Present
Auto	1 to 9	93.28	97.29	0	TRUE

Cement	1 to 9	587.18	97.29	0	TRUE
Foods	1 to 9	3517.77	97.29	0	TRUE
Textile	1 to 9	2527.54	97.29	0	TRUE
Banks	1 to 9	507.76	97.29	0	TRUE
Pharmaceutical	1 to 9	3235.86	97.29	0	TRUE
Chemicals	1 to 9	587.18	97.29	0	TRUE
Oil and Gas	1 to 9	89.83	97.29	0	TRUE

Table 3. White noise test

Sector	Lag	Score	C.V.	P Value	Pass?
Pharmaceutical	1 to 9	1964.08	97.29	0	FALSE
Auto	1 to 9	1510.01	97.29	0	FALSE
Oil & Gas	1 to 9	1205.06	97.29	0	FALSE
Cement	1 to 9	379.45	97.29	0	FALSE
Food	1 to 9	649.59	97.29	0	FALSE
Textile	1 to 9	224.39	97.29	0	FALSE
Banks	1 to 9	5932.58	97.29	0	FALSE
Chemical	1 to 9	829.9	97.29	0	FALSE

A normality test was developed by Jarque and Bera (1980, 1987) to determine whether the sample data's skewness and kurtosis are consistent with a normal distribution. The test is typically applied to determine whether a series has a normal distribution by putting the null hypothesis into the test. The series is deemed to have a normally distributed distribution if the skewness is equal to and the kurtosis value is 3. All eight sectors were subjected to the normality test. The findings of Jarque-Bera disprove the normality hypothesis across all industries.

Table 5. Test of Stationarity

Sector	DF-statistics	Lag order	*p* Value
Banks	-31.21	1.00	0.00
Textile	-28.05	1.00	0.00
Foods	-32.69	1.00	0.00
Cement	-31.88	1.00	0.00
Oil & Gas	-33.10	1.00	0.00
Chemicals	-29.86	1.00	0.00
Auto	-29.84	1.00	0.00
Pharmaceutical	-28.23	1.00	0.00

The unit root test or stationary test is used to obtain reasonable results on time series data (Gujarati, 1995). The augmented Dickey Fuller (ADF) test is used to test the stationarity of financial time series data. The ADF test results confirm that the time series is stationary (p value <0.05).

Table 6. Jarque-Bera normality test

Sector	Score	C.V.	P Value	Pass?
Pharmaceutical	67.64	5.99	0.00	FALSE
Auto	2376.78	5.99	0.00	FALSE
Chemicals	611.85	5.99	0.00	FALSE
Oil & Gas	9124.85	5.99	0.00	FALSE
Cement	82.36	5.99	0.00	FALSE
Food	303.33	5.99	0.00	FALSE
Textile	6377.69	5.99	0.00	FALSE
Banks	339.58	5.99	0.00	FALSE

5. Empirical Results

To investigate the relationship between the sector returns of eight sectors of the Pakistan Stock Market, we employed biwavelet analysis using the CWT method to capture the strength of the interaction between two time series of data across the time and frequency domains. The thick black curve in the graph indicates the significance levels of 5% and 10%. The cone of influence (COI) shows significant relationships or comovements, and beyond the COI are unreliable comovements of time series data **(Grinsted et al., 2004)**.

5.1 Two-pair scalograms

The wavelet coherence graphs are given in pairs between the sectors of banks, textiles, autos, pharmaceuticals, chemicals, oil and gas, and cement. In this figure, the horizontal axis denotes time, and the vertical axis symbolizes the scale. The frequency is changed to time units to make comprehension easier (days). The figure is composed of color codes, each of which has a distinct meaning. For instance, blue denotes low correlations, which indicate a relationship between two time series that lasts one week. The two series are tightly connected, as shown in red, which indicates high correlations. From blue to red, the color coding depicts a power spectrum.

Wavelet analysis also identifies whether the comovement is strong or low through time and between frequencies, possibly capturing different aspects of the interaction between the two in terms of time and frequency, banks, and the textile couple. The area where the comovement between the two series is statistically significant is delineated by a thick black line at a significance level of 5%. The arrows depict the phases of the two time series. The direction of the arrows represents the phase difference between two time series. Arrows pointing to the right (left) or up (down) denote phases of the variables; arrows pointing to the right and up (down) denote leading (lagging); and arrows pointing to the left and up (down) denote lagging (leading). Several computations were carried out using the R programming language.

The wavelet plot in Fig. 5.1 shows a weak link between banking sector returns and the textile industry over the preceding 10 years because of the short retention periods of 2 to 4 and 4-8 days, which provide options for diversification. The dark and light blue regions of the banking and textile pair are highlighted in the figures for medium retention periods of 16-64 and 64-128, respectively. The pair is favorable for diversification over the medium term, despite the presence of some red and yellow zones. The coherence plot also reveals less correlation for long-term retention periods between 128 and 256 days, which can provide investors with the greatest gains from diversification.

Wavelet plots for the sectors of banks and pharmaceuticals are shown in Figure 5.2. There is a poor correlation between the returns of the banking sector and the pharmaceutical industry in this pair over short retention periods of 2 to 4 and 4-8 days, respectively, providing options for investment diversification. The years 2015, 2018, and 2019 are somewhat connected but still provide some benefits of diversity for the medium retention period between 16 and 64 and between 64 and 128. The coherence figure demonstrates a substantial correlation for holding durations longer than 128–256 days, which reduces the benefits of diversification. In contrast to long-term retention periods, short- and medium-term investments provide investors with more potential rewards.

The wavelet plots for the banking and chemical sectors are shown in Figure 5.3. For the 2–4 and 4–8-day short retention periods, banking sector returns over the preceding ten years have shown a minimal link to the chemical industry, which presents potential for investment diversification. The coherence figure for the medium retention periods (16-64 and 64- 128) shows numerous red and yellow patches with high correlations. The benefits of investing in this pair for a medium-term retention period may not be favorable to investors. The plot also shows indications of substantial comovement during a long holding time of 128-256 days. Due to the high correlation, the long retention period also provides fewer diversification benefits.

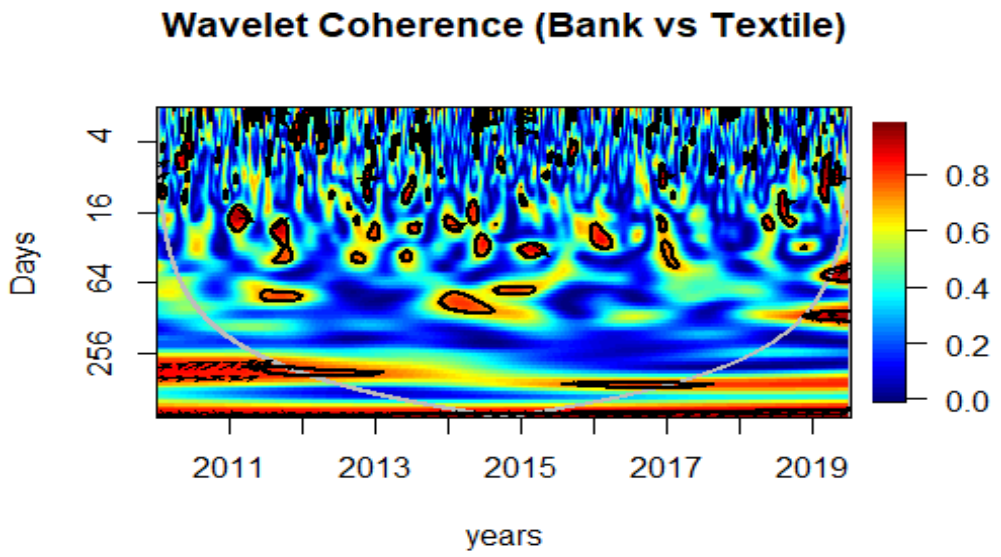
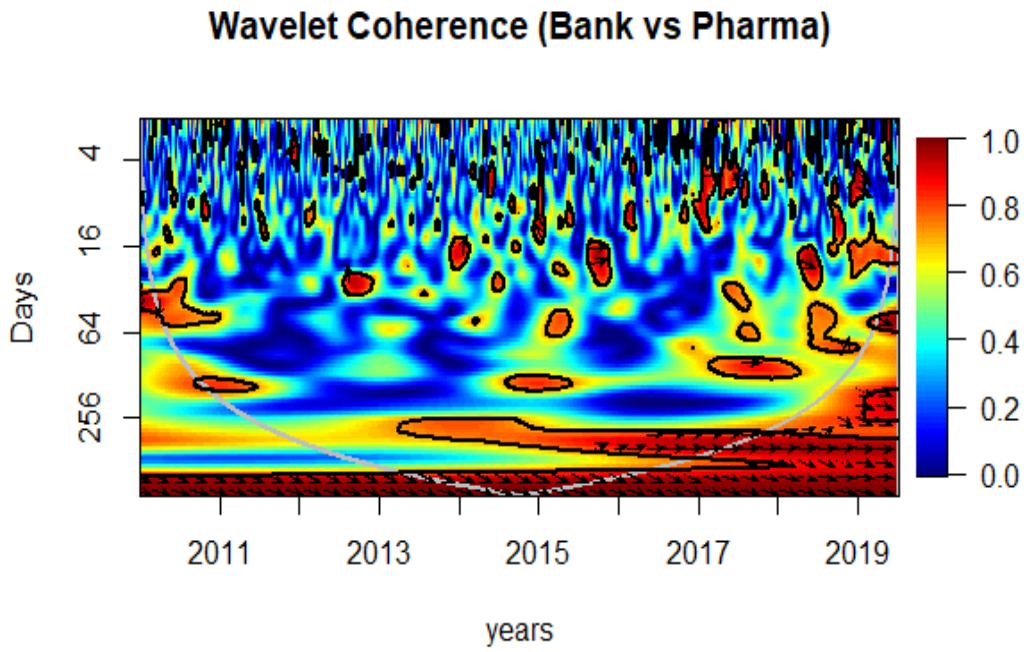


Figure 4.1 Wavelet Coherence Banks vs. Textile



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294 **Figure 4.2 Wavelet Coherence Bank v/s Pharma**

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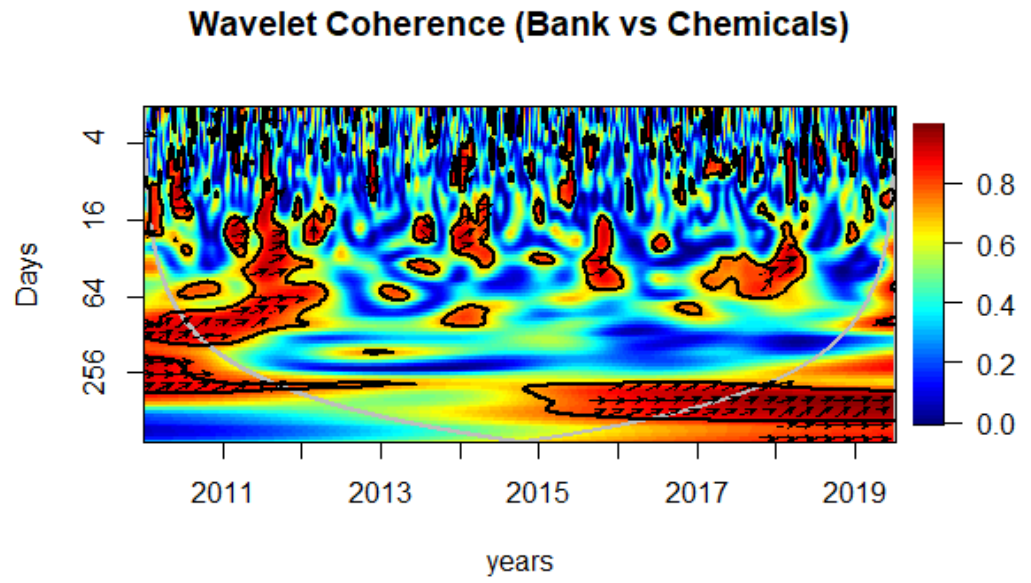

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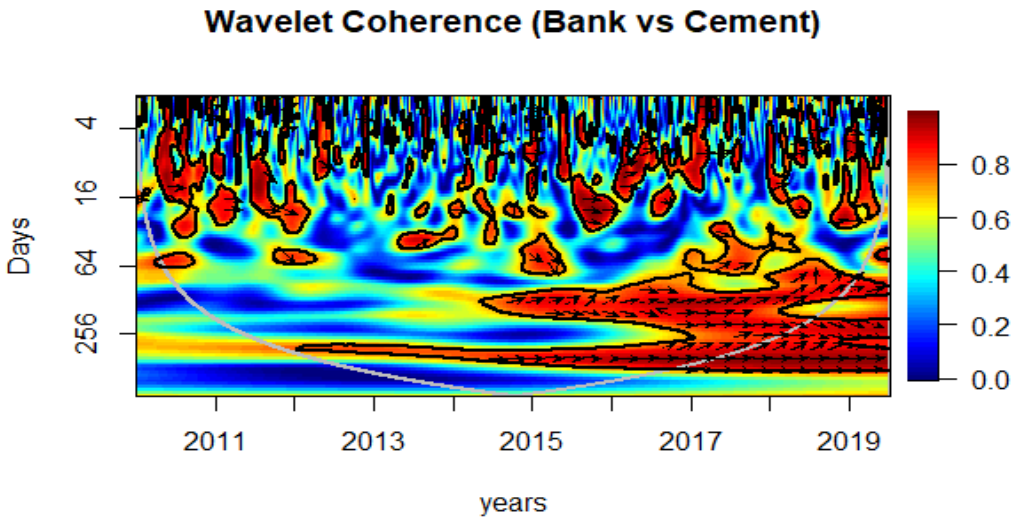
297 **Figure 4.3 Wavelet Coherence Banks vs. Chemicals**



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299 **Figure 4.4 Wavelet Coherence Banks v/s Cement**

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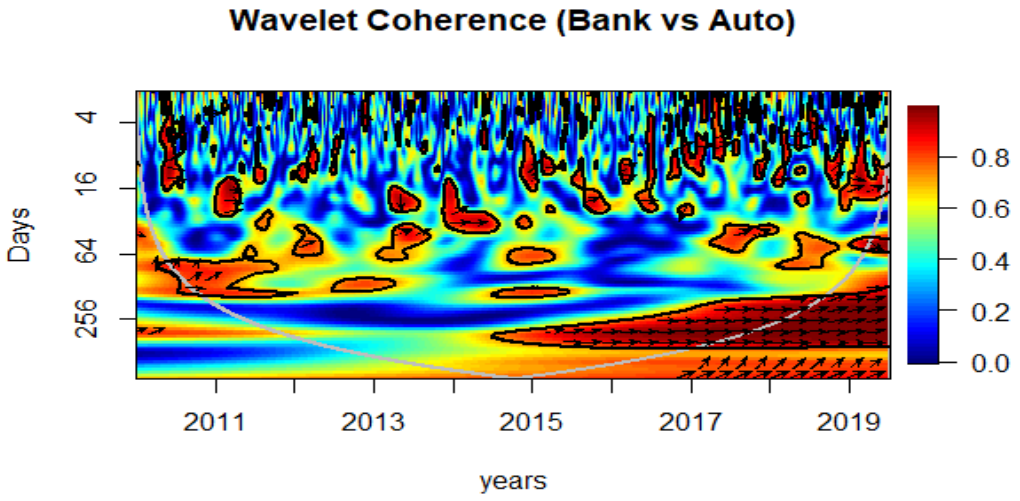

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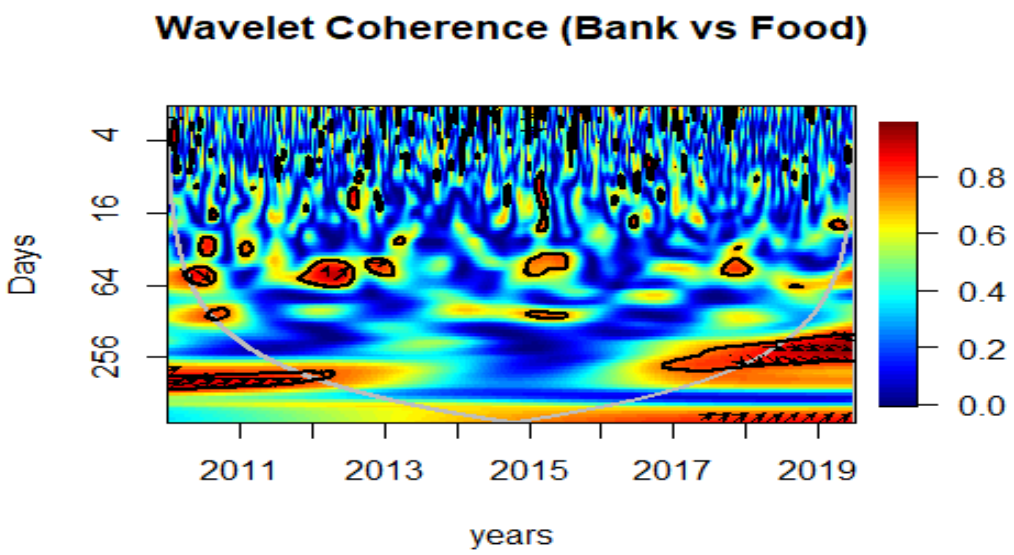
302 **Figure 4.5 Wavelet coherence banks v/s Auto**



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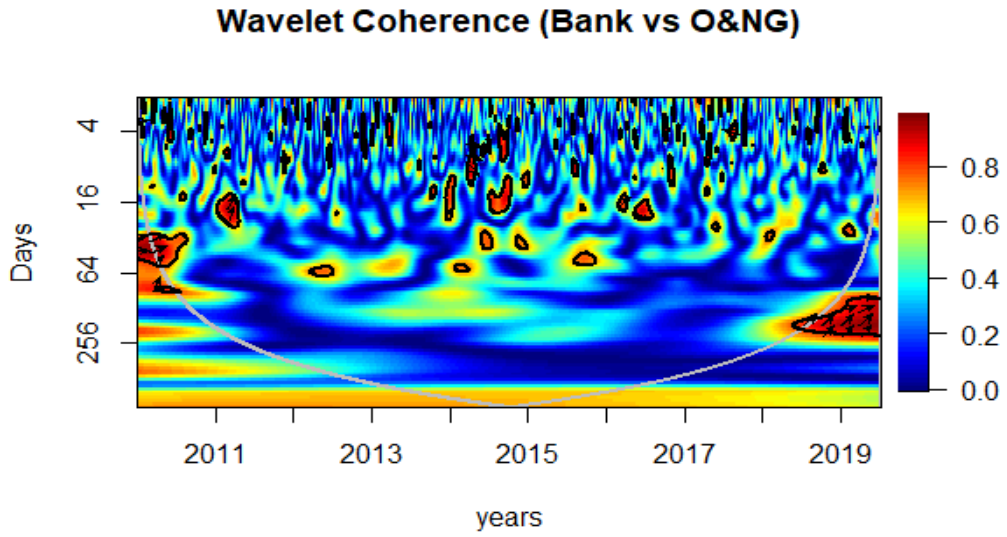
304 **Figure 4.6 Wavelet Coherence Banks vs. Food**



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306 **Figure 4.7 Wavelet Coherence Banks vs. Oil and Gas**

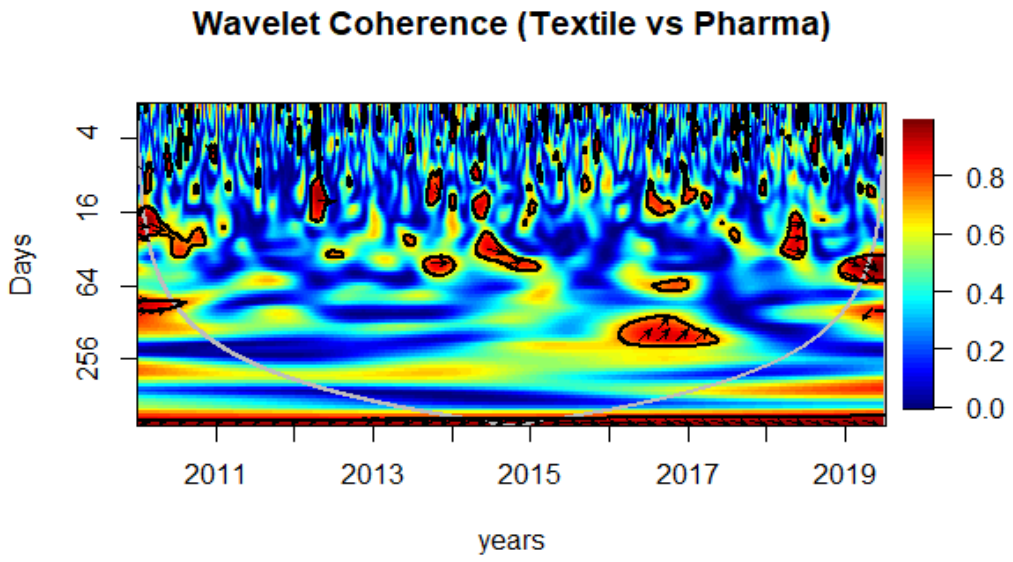


Figure 4.8 Wavelet Coherence Textile v/s Pharma

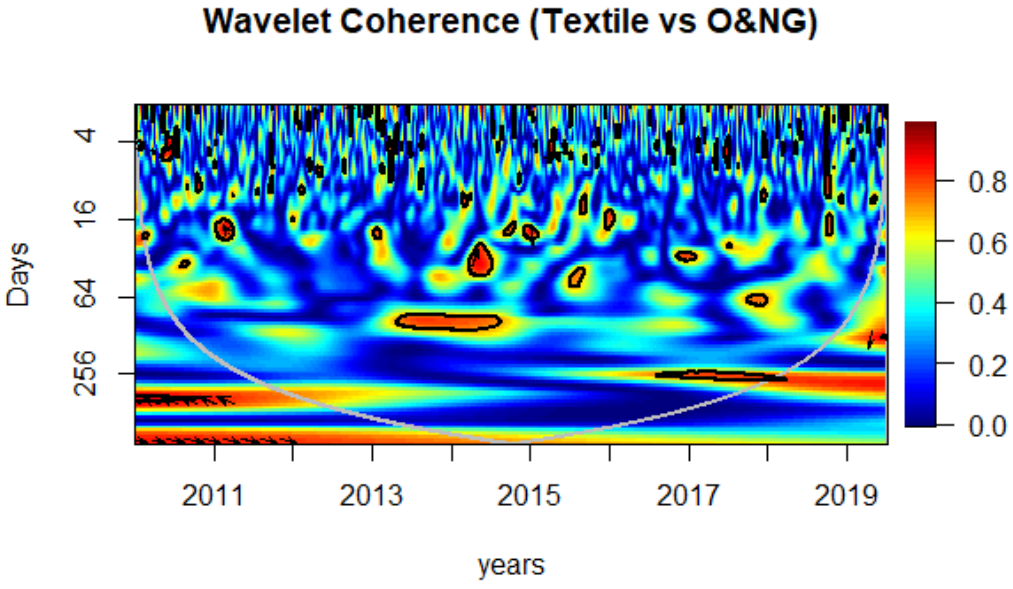
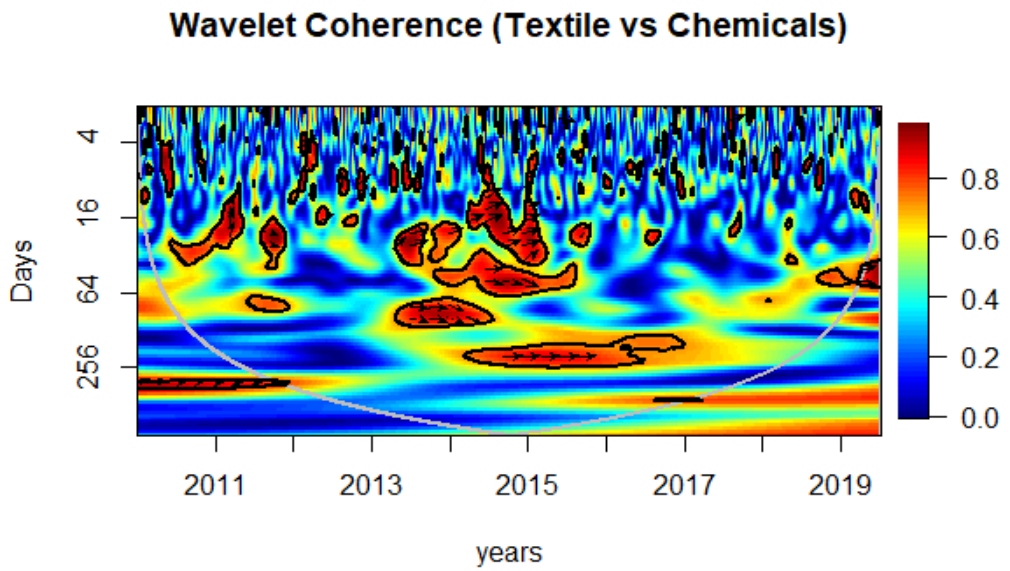


Figure 4.9 Wavelet Coherence Textile vs. Oil and Gas



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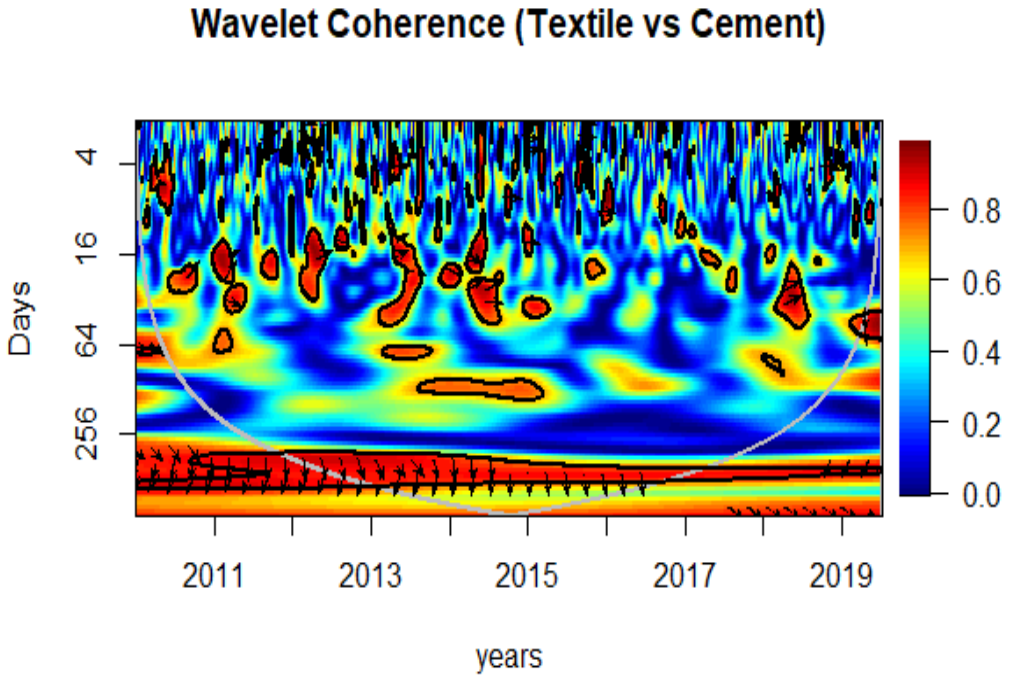
312 **Figure 4.10 Wavelet Coherence Textile v/s Chemicals**



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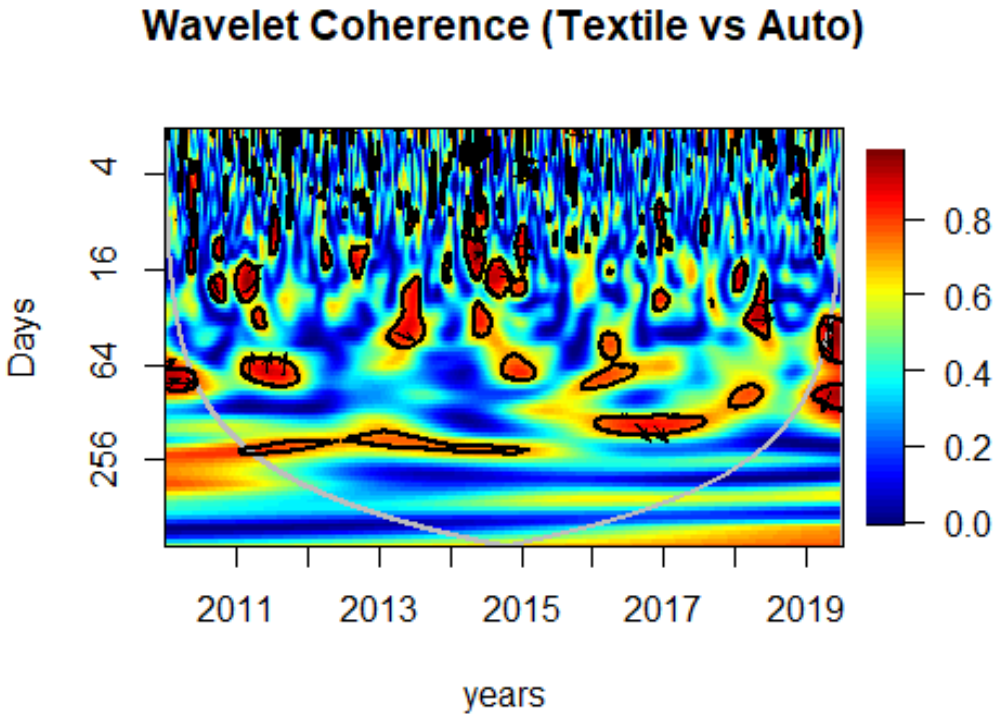
314 **Figure 4.11 Wavelet Coherence Textile vs. Cement**



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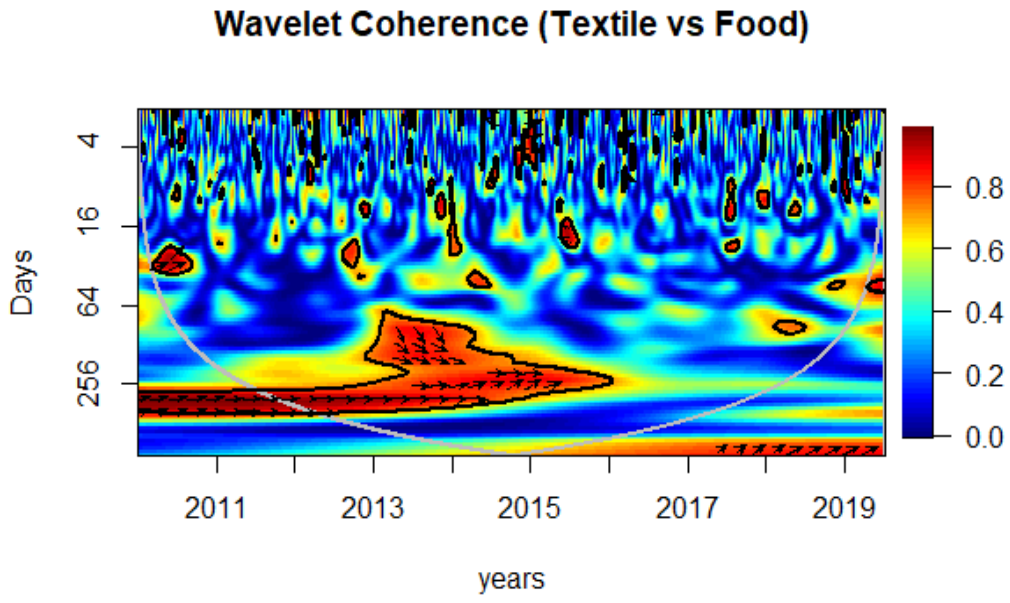
316 **Figure 4.12 Wavelet Coherence Textile v/s Auto**



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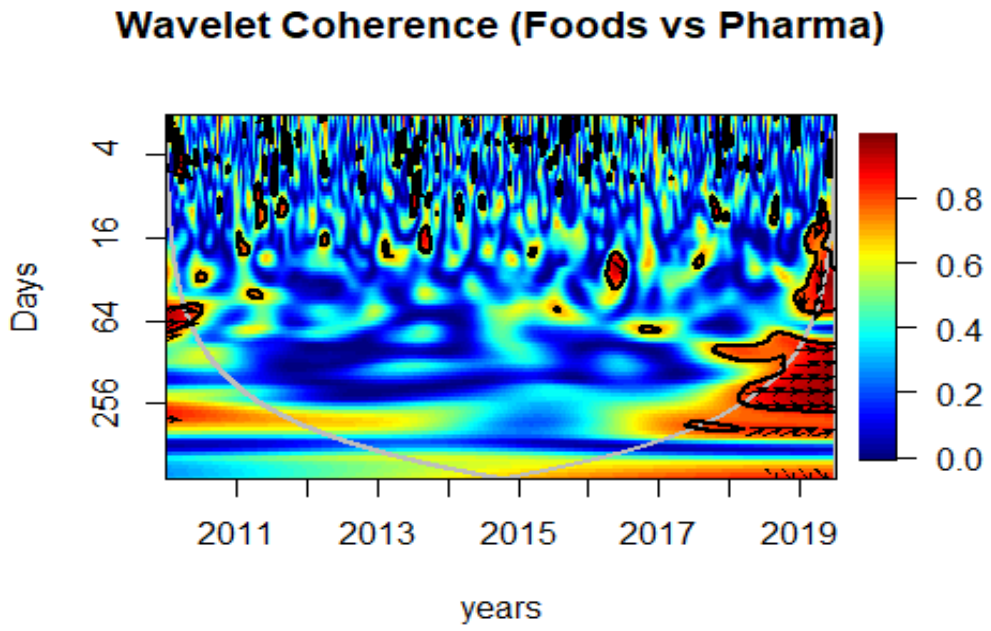
318 **Figure 4.13 Wavelet Coherence Textile vs. Food**



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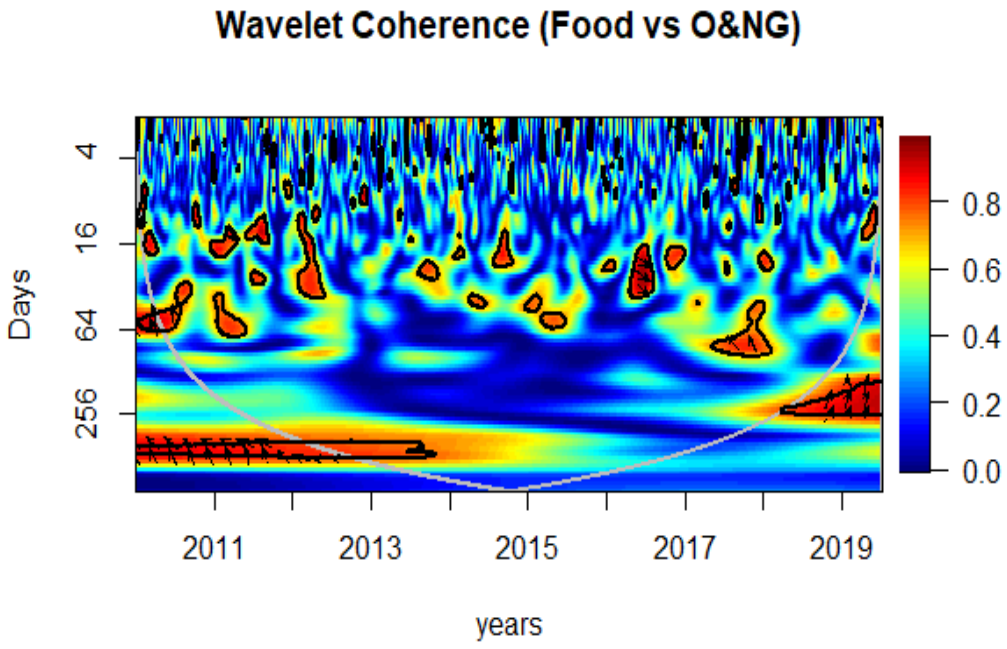
320 **Figure 4.14 Wavelet Coherence Food v/s Pharma**



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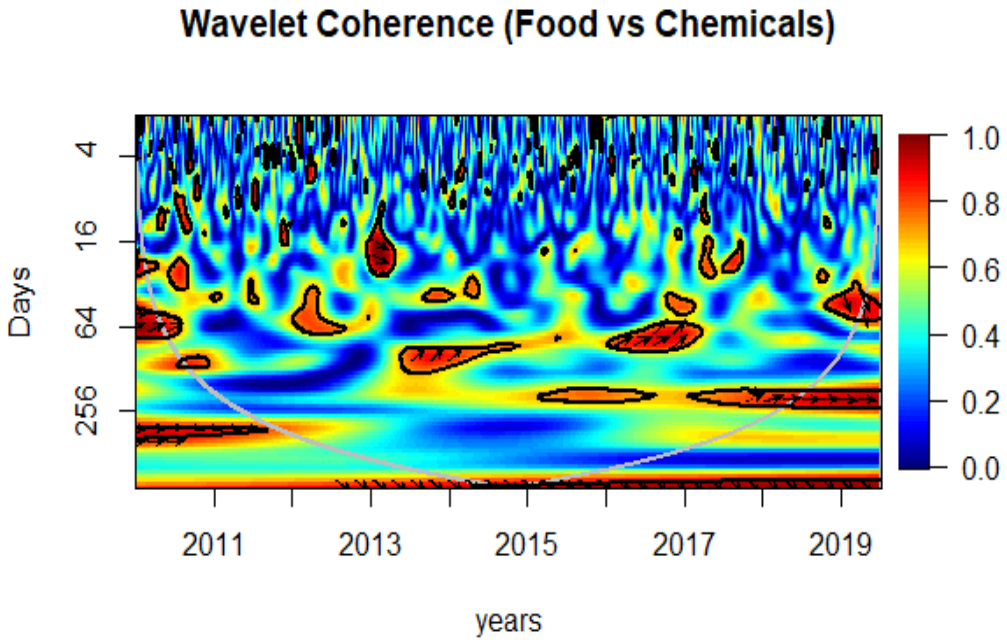
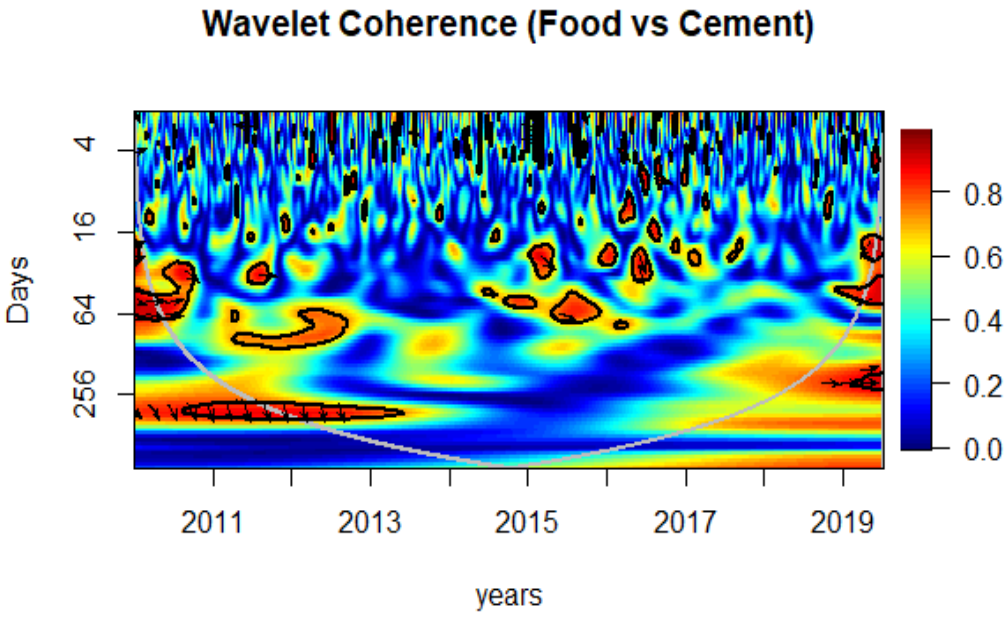
322 **Figure 4.15 Wavelet Coherence Food vs. Oil and Gas**



323

324 **Figure 4.16 Wavelet Coherence Food v/s Chemicals**



325

Figure 4.17 Wavelet Coherence Food vs. Cement

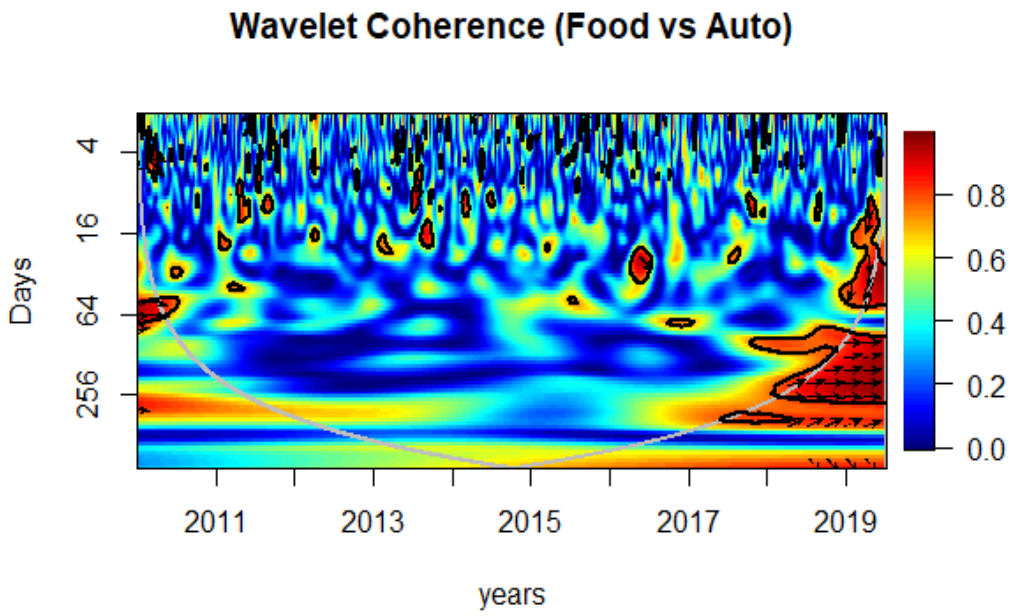


Figure 4.18 Wavelet Coherence Food v/s Auto

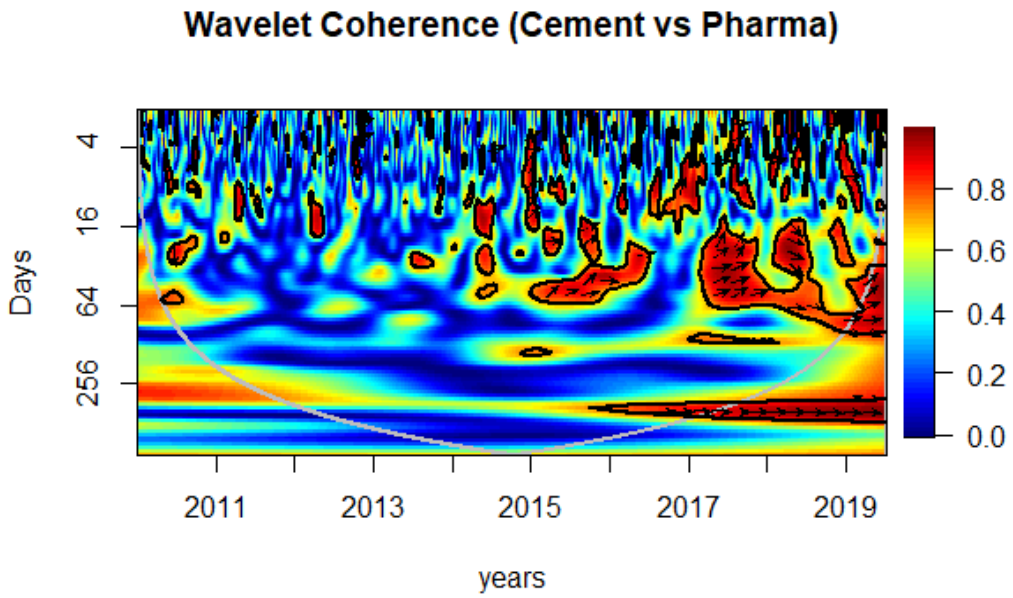


Figure 4.19 Wavelet Coherence Cement v/s Pharma

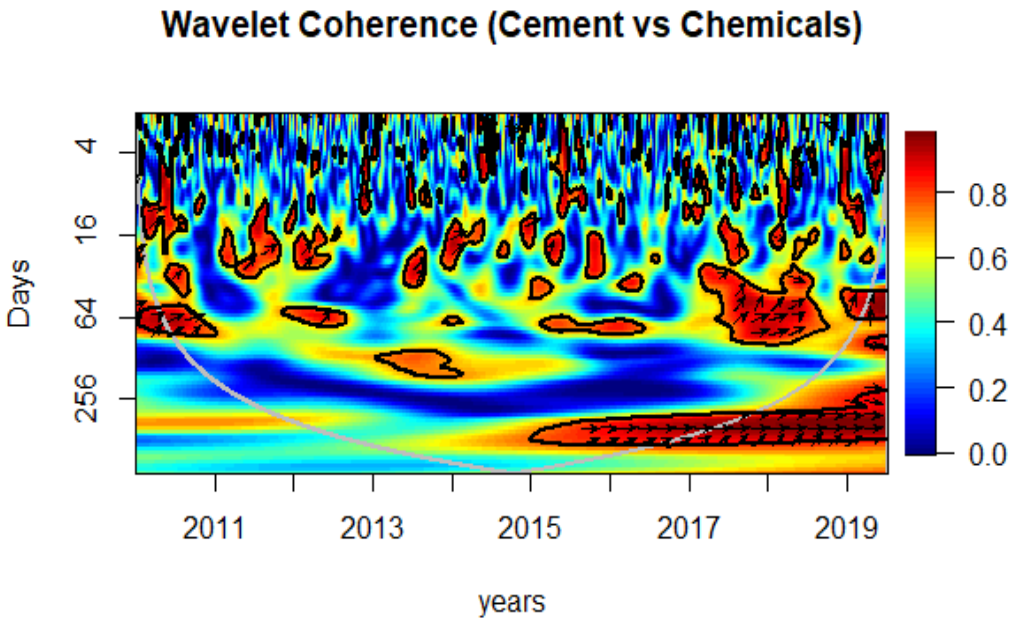


Figure 4.20 Wavelet Coherence Cement vs. Chemicals

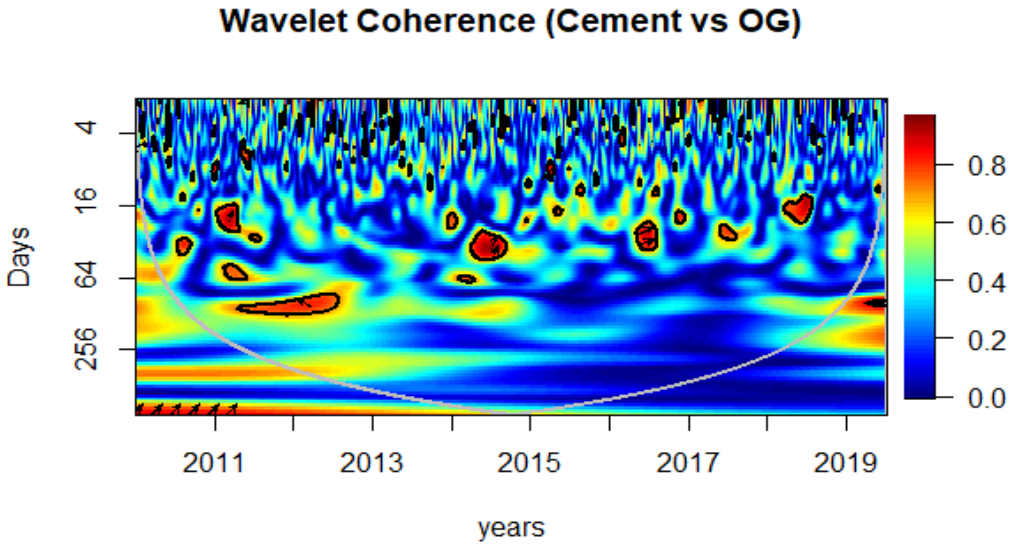


Figure 4.21 Wavelet Coherence Cement vs. Oil and Gas

Figure 4.38 presents the wavelet coherence plots of the cement and auto pair of sectors. In this pair, for short-term periods ranging from 2 to 4 days and 4 to 8 days, the comovement was weak over the sample period of 10 years. are highlighted in blue in the diagram. It offers the opposite for portfolio diversification. However, for the medium-term term periods from 16 to 64 days and 64 to 128 days, the correlation is also weak, with some variations, as indicated by the red and yellow regions, respectively. Investors can also choose to invest in the medium term. However, for long-term holding periods ranging from 128 to 256 days, the pair is weakly correlated, which is a favorable option for investment. Investors are encouraged to diversify their investment portfolios at all time scales from short-term to long-term holding periods.

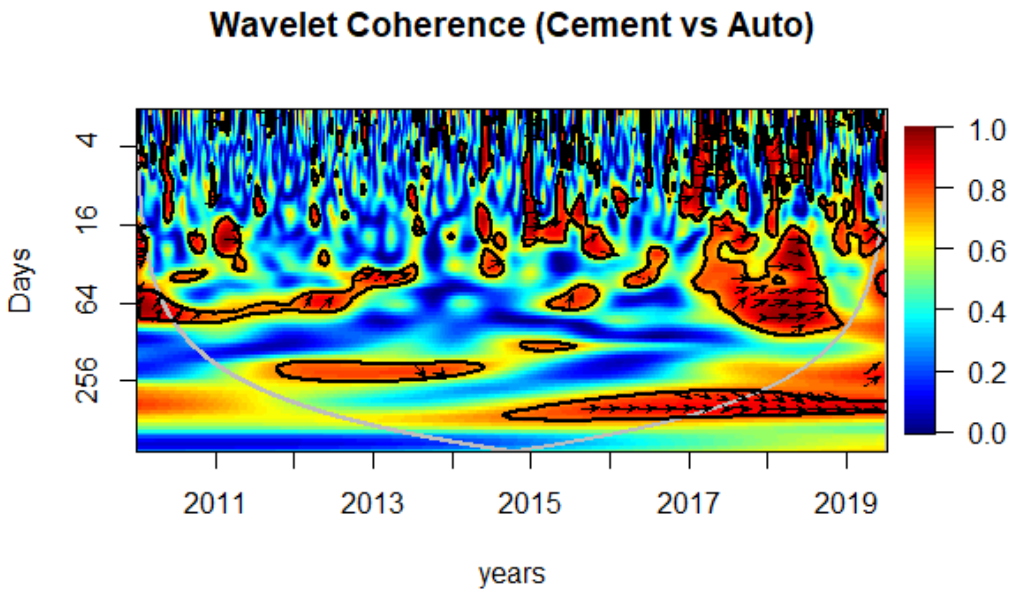


Figure 4.22 Wavelet coherence cement v/s auto

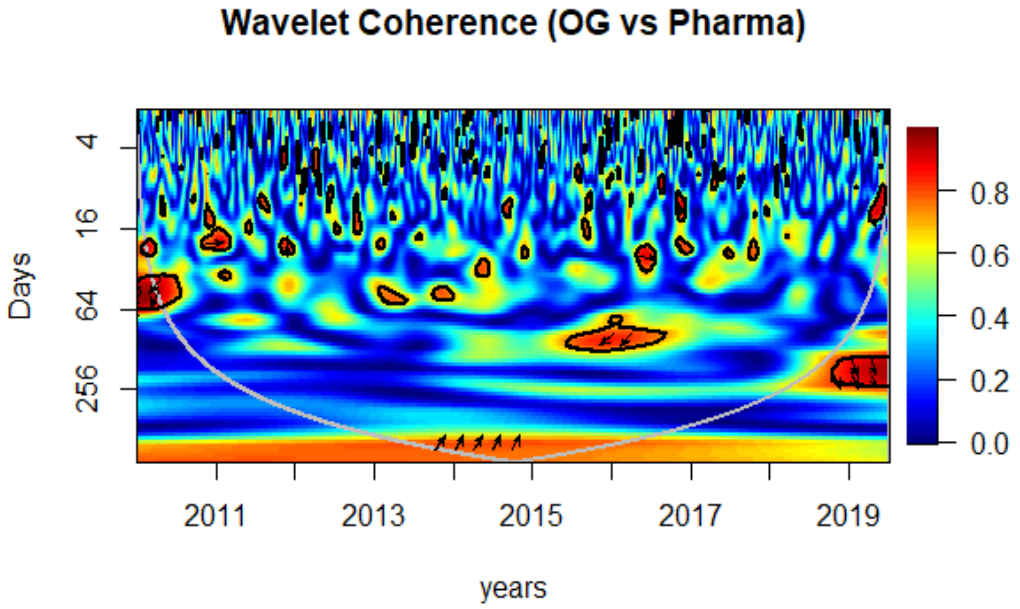


Figure 4.23 Wavelet Coherence Oil & Gas v/s Pharma

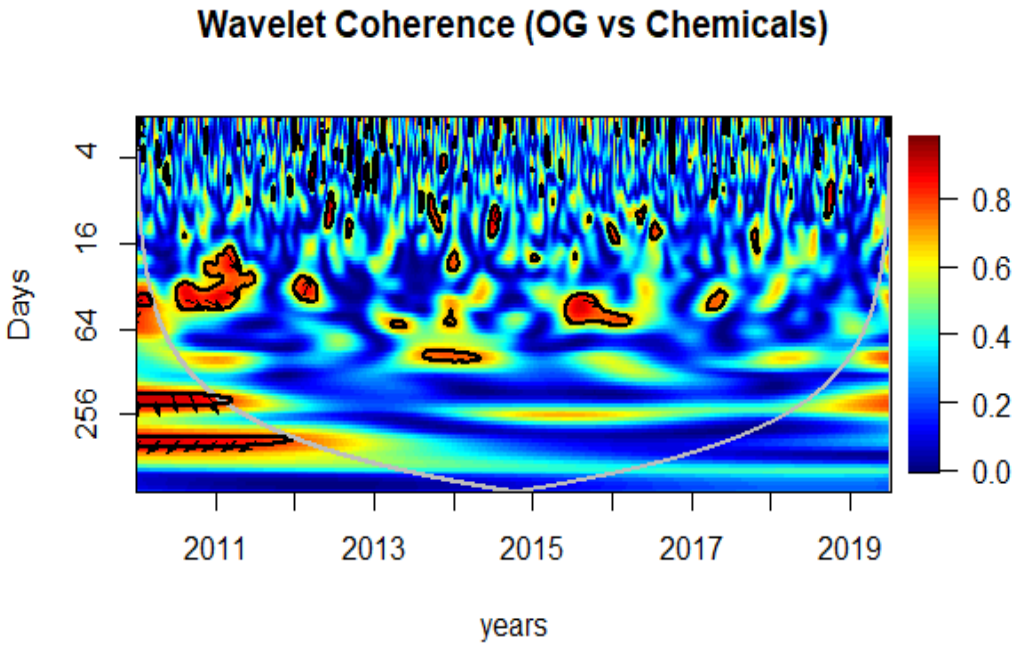


Figure 4.24 Wavelet Coherence Oil & Gas v/s Chemicals

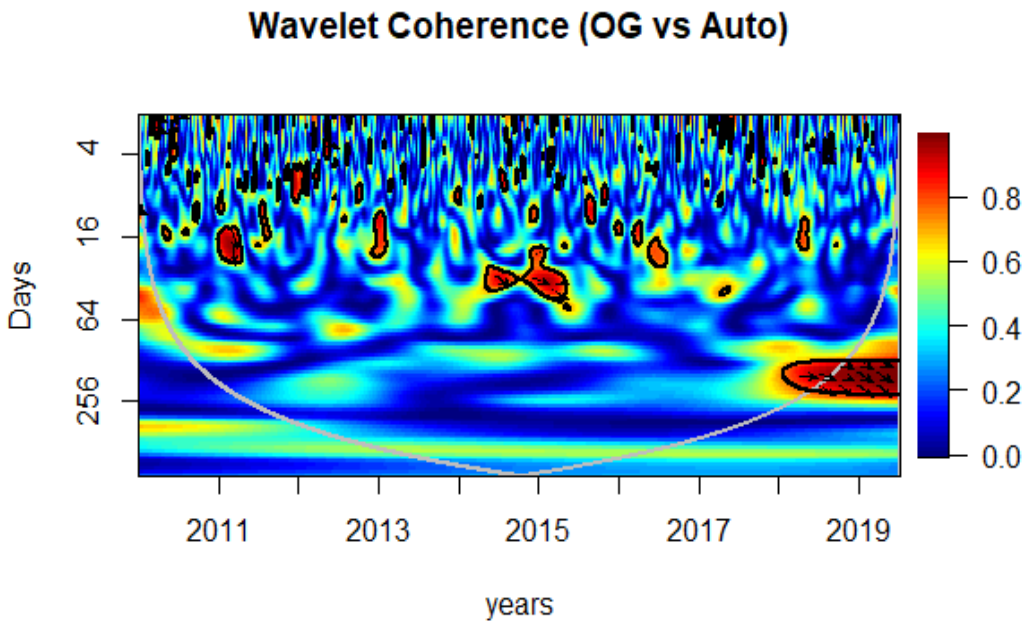


Figure 4.25 Wavelet Coherence Oil & Gas v/s Auto

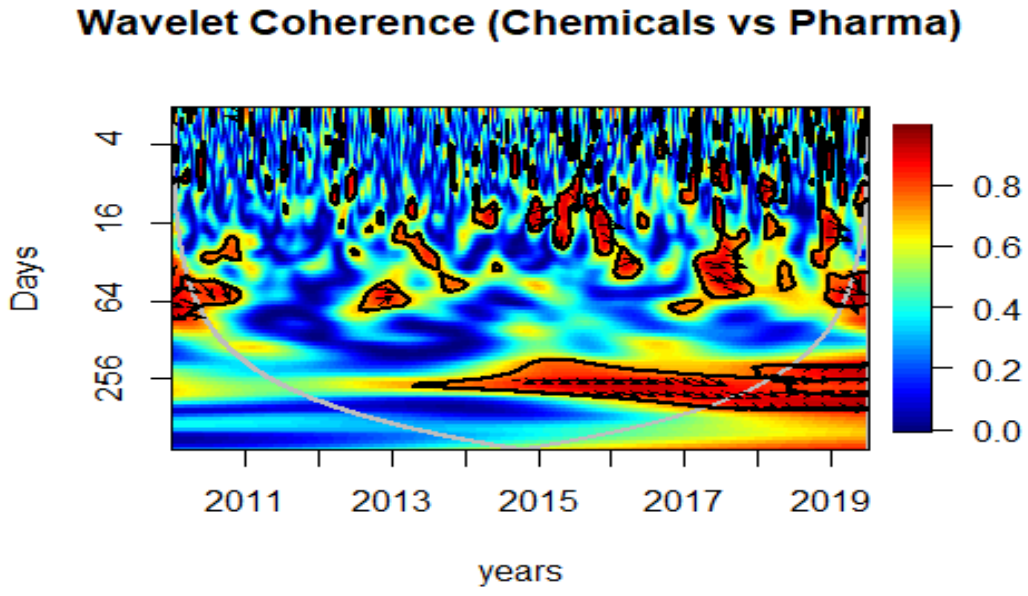


Figure 4.26 Wavelet Coherence Chemicals v/s Pharma

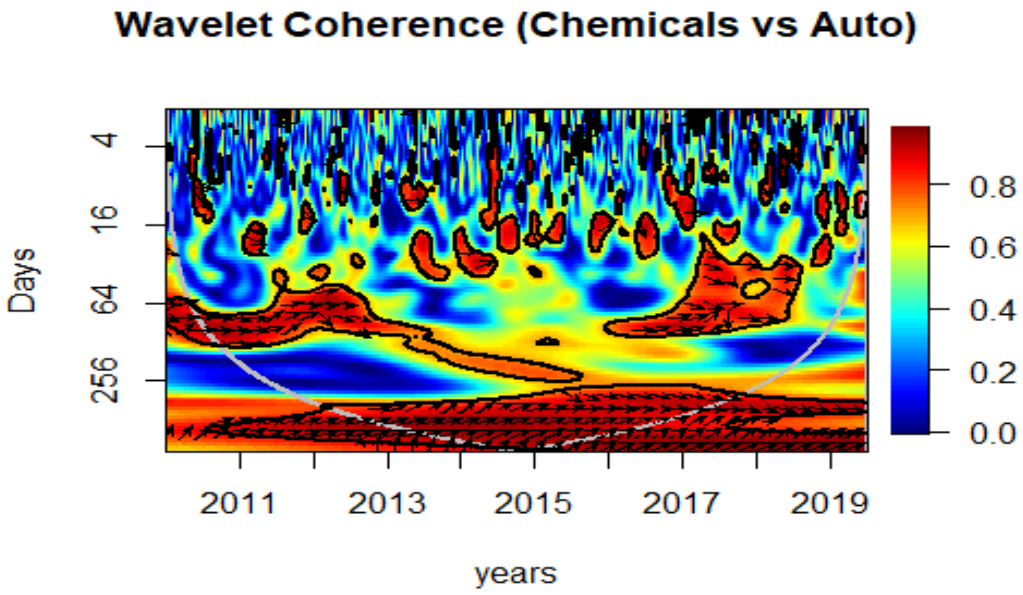


Figure 4.27 Wavelet Coherence Chemicals v/s Auto

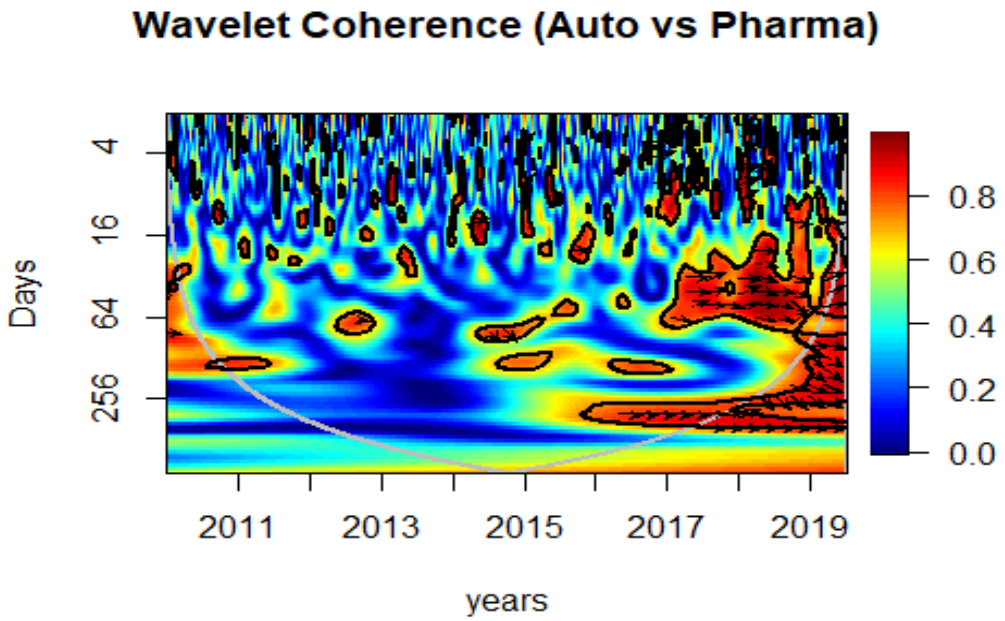


Figure 4.28 Wavelet Coherence Auto v/s Pharma

The figure shows a wavelet plot of the bank and cement pair of sectors. In this pair, for almost all the retention periods, from the shorter retention periods of 2 to 4 days and 4-8 days to the medium-term retention periods of 16-64 and 64-128 and the long-term retention period ranging from 128-256, a high correlation was detected due to the presence of many red and yellow regions in the plot. The correlation seems to be greater in this pair than in the other pairs discussed above. This pair may not offer favorable opportunities for diversifying the investment due to high correlations. Therefore, investors are not advised to invest in this pair.

Due to these significant correlations, banks and auto industry pairs might not benefit from portfolio diversification during medium- and long-term retention periods. The reduced link between banks and the food industry suggests profitable opportunities for investment diversification. Due to lower correlations, this pair generally provides better options for portfolio diversification. By making medium-term investments, portfolio diversification can also be achieved between banks and the oil-gas sector. However, with low correlation, as indicated by the light blue and blue regions in the plot, long-term comovement (128-256 days) is likewise only weakly associated; therefore, holding the shares of these two pairs in the long term will likely provide diversification benefits.

The wavelet plots of the textile and pharmaceutical sectors are shown in Figure 4.25. Due to poor correlation, as shown in the plots with more blue and light blue regions, this pair provides the benefits of diversification during short retention periods, such as 2-4 and 4-8 days. Although the correlation appears to be slightly greater for the medium-term retention periods of 16–64 days and 64–128 days in 2014 and again in 2017, it may also have some advantages for portfolio diversification. The textile sector is less associated with pharmaceuticals for long-term investments with 128–256-day retention periods, which is highlighted in the coherence diagram. Throughout the long-term retention period, this pair also offers some opportunities for portfolio diversification.

Oil and gas industry and textiles: The diagram highlights some red and yellow regions for the medium-term retention period, which is between 16-64 days and between 64-128 days, which is suggestive of some correlation but may be negligible due to the presence of blue and light blue regions, which constitute the majority of the space. The pair is also weakly correlated for the long-term retention period, which ranges from 128-256 days, providing appropriate options for portfolio diversification.

Wavelet plot of the pair of sectors for textiles and chemicals. Over the short retention periods of 2-4 and 4-8 days in this pair, the textile sector is only moderately connected to the chemical sector, highlighting the advantages of portfolio diversification. However, as shown by the red areas with arrows and the yellow region, the nature of comovement in both the medium-term and long-term holdings appears to be greater. It is not advised to diversify investments in these two areas over the medium and long term.

Sector wavelet plot for the textile and cement industries Due to the presence of blue and light blue regions in the plot over short-term flooding periods of 2-4 days and 4–8 days, this pair is also weakly associated. Diversification has advantages that might be obtained by investing during a brief retention period. However, in the medium-term retention periods of 16-64 and 64-128 days, the movement appeared to be mild. The long-term holding duration of this pair is highly coupled, providing few opportunities for portfolio diversification.

Textile and autoindustry wavelet plots showing the relationships between the two sectors. Over the past ten years, the textile sector has shown less correlation with the auto industry across

short time frames, such as 2-4 days and 4-8 days, and has benefited from portfolio diversification. The red and yellow sections, however, show that the correlation is marginally greater during the medium-term retention period. The advantages of diversification are also achievable over the long run. The pair exhibits some red regions during the long-term retention period, but the comovement is mild, making it a good alternative for investment diversification.

Diversification advantages in textiles and food can be attained in a brief retention period. From 16-64 days, the comovement is less for the medium-term retention period, but from 64-128 days, it is stronger from 2013 to 2015. Holding an investment for 16–64 days is possible. Last, the correlation is larger for the long-term period from 128-256, indicating that investors cannot gain from diversification. It is advised that investors invest for a short- to medium-term holding duration of no longer than 64 days.

For the textile vs. food combination, the coefficient of friction is reduced in the short term (2–4 days) and long term (4–8 days). Investors might reap the short-term rewards of diversifying their investment portfolio. The food sector is also less correlated over the medium-term retention periods of 16-64 and 64-128 days, which is a good alternative for investment diversification. Finally, due to the existence of blue and bright regions in the plot, the long-term holding time of 128-256 days appears to be less associated with some variance in 2018 and 2019. Because investors can maximize their gains during all stock retention periods, from the short-term to the long-term, this pair presents better investing opportunities.

The wavelet plots of the food vs. oil & gas pair of sectors are shown in the food vs. oil & gas sector graph. Similarly, for the preceding ten years, this pair had a weak correlation across short retention periods of 2-4 and 4-8 days. This provides opportunities for short-term investment diversification. The red and yellow portions of the plot, which provide lower benefits, indicate that the comovement appears to be slightly greater for the medium-term retention periods of 16-64 and 64-128 days, respectively. While there is little link between long-term retention periods of 128-256 days and certain variances in 2012 and 2013, these periods provide the benefits of diversification.

The wavelet plots of the food and textile sectors are shown in the food vs. chemicals section. Within the brief time frames of 2-4 days and 4-8 days, the food sector in this pair has a weak association. It is advised that investors make short-term investments. The comovement is slightly greater in the medium-term retention period, which spans from 16-64 and 64-128 days, as indicated by the numerous red and yellow patches in the diagram, but advantages from diversification may be made in the medium-term retention period. The comovement is likewise moderate over the long-term period between 128 and 256 days, which is advantageous for investment.

The wavelet plots of the food and cement sectors are shown in the article Foods vs. Cement. Over the 10-year data period, this pair has a weak correlation with short retention periods of 2-4 and 4-8 days. It offers favorable investment possibilities that allow investors to reap the rewards of diversity. The medium-term period, however, between 16-64 and 64-128 days, shows some red and yellow zones but is still conducive to diversification. The comovement is also minimal in the long-term retention period from 128-256 days, with modest changes from 2011 to 2013. It is advised that investors buy this pair for retention periods ranging from short to long.

The wavelet plots for the food and automobile sectors are shown in the food and automobile industry section. In this pair, the short-term periods of 2-4 days and 4-8 days across the 10-year data period show a poor correlation between the food sector and the car sector. It provides opportunities to diversify investments and generate returns on investments. The correlation appears to have decreased over the medium-term periods of 16-64 days and 64-128 days, which is also

encouraging for investment. Last but not least, the food industry also provides the advantages of diversification in the long-term holding duration of 128-256 days due to decreased comovement in the pair with some variances in 2018 and 2019. Investors are advised to purchase this pair for all retention periods.

The wavelet plots of the cement and pharmaceutical sectors are shown for cement vs. pharmaceutical. With minor variance from the start until 2015 across the data period of ten years, this pair reveals good associations in the short-term periods from 2 to 4 and 4-8 days. The comovement is smaller for medium-term retention periods, such as 16–64 and 64–128 days, until 2014 but grows after that year until 2019. With some variance in 2016 and 2017, the comovement is less for the long-term period of 128-256 days than for the medium-term period. By making investments with a long retention period, investors can benefit from doing so.

The wavelet coherence graphs of the cement and chemical pairs of sectors are shown in Cement vs. Chemicals. Over the sample period of 10 years, this pair exhibited less comovement in the short-term ranges of 2-4 and 4-8 days. In the short term, it might provide the advantages of diversification. The nature of comovement is the same for retention periods that are mild to lengthy. Both the medium-term retention period of 16–64 days and the long-term retention period of 64–128 days had somewhat greater comovement. Due to greater comovement, the benefits of diversification can be diminished.

The wavelet coherence charts for the cement and automobile sectors are shown in Cement vs. Oil and Gas. The blue color in the diagram indicates that there is weak comovement in this pair of short-term periods from 2-4 and 4-8 days across the sample period of 10 years. It provides opportunities for diversifying a portfolio. However, as shown by the red and yellow regions, the association is also weak, with some fluctuations for the medium-term time periods of 16-64 days and 64-128 days. Investors have the option of making medium-term investments as well. However, the pair is weakly correlated during a long retention period of 128-256 days, making investments in this period more attractive. It is advised that investors diversify their investment portfolios throughout all retention periods, from the short to long term.

The wavelet coherence graphs of the Cement and Auto pair of sectors are shown in Cement vs. For the 10-year data period, the comovement in this pair is greater during the short-term retention periods of 2-4 and 4-8 days. Due to the abundance of the red and yellow regions in the diagram suggesting vulnerability, the correlation seems to be highest for the medium-term retention periods, such as 16–64 days and 64–128 days. Additionally, this pair exhibits a strong correlation over a long holding time of 128–256 days. It is not advised for investors to purchase this pair.

The wavelet coherence charts for the two industries of oil and gas versus pharmaceuticals are displayed. During the course of the ten-year study period, there was a weak correlation between these pairs when held for brief periods of 2-4 and 4–8 days. It provides the chance to diversify investment portfolios. However, the comovement is also mild, with some variations for the medium-term periods of 16-64 and 64-128 days. However, the pair exhibited minimal association over the long-term retention period of 128-256 days. By purchasing this pair, investors may also gain.

The wavelet coherence graphs of the Oil & Gas and Chemicals pair of sectors are shown in Oil & Gas and Pharama. Additionally, across the sample period of 10 years, this pair exhibits a weak correlation in brief periods of 2-4 and 4-8 days. Short-term investment portfolio diversification is possible. The correlation decreases with some variance for the medium-term retention periods of 16–64 days and 64–128 days, which can be advantageous for investment. In

contrast, the comovement appears to be weaker throughout the long-term retention period of 128-256 days, indicating that investors can maximize their profits on investment in this pair.

A wavelet plot of oil and gas with an auto pair of sectors is shown for oil and gas with an auto pair. In this pair, a modest association is depicted in the plot over the sample period of 10 years for short-term retention periods, such as 2-4 days and 4-8 days. There are numerous benefits of investing in diversification in the near term. However, the pair is also less correlated, with some fluctuations during the medium-term retention periods of 16–64 and 64–128 days, providing opportunities for diversification. Finally, this pair likewise exhibits poor correlation over the long-term retention period from 128-256. In general, this pair is appropriate for diversifying the investment.

Chemicals and pharmaceuticals display wavelet coherence graphs for two industries: chemicals and pharmaceuticals. In the two- to four-day and four- to eight-day short-term time frames, this pair has a weak correlation. Short-term investment horizons can reap the rewards of diversification. However, it appears that the form of the comovement is the same in all three medium-term retention periods—16-64 days, 64-128 days and 128-256 days—indicating a strong association. Straight arrows pointing to the right indicate a positive association. A short-term holding duration is advised for investments, whereas medium- and long-term retention periods are discouraged.

The wavelet plots of the chemical and auto sectors are shown in the chemical vs. auto industry graph. The red and yellow portions of the plot show that the above diagram and the sample period of 10 years have a strong correlation. The degree of coupling is quite powerful. There are extremely few possibilities for diversifying investments from short- to long-term retention periods. Investors are not advised to make investments in these two areas.

The wavelet coherence charts for the Auto and Pharmaceutical pair of industries are shown in Auto and Pharmaceutical. In this pair, the degree of comovement is weak across the 10-year data period for short-term holiday periods of 2-4 days and 4–8 days. In the near future, investors may diversify their investment holdings. After 2017 to 2019, the correlation is greater for the medium-term retention period. In the extended retention period, the link appears to be weaker until 2016, but it is stronger until 2019. In the long-term retention period, this combination can provide some benefits, such as partial diversification.

Table 6. Investment Advice

Short Term (2-4 & 4-8 days)	Medium Term (16-64 & 64-128)	Long Term (128-256 days)	Investment not recommended
Banks,Textile	Banks,Textile	Banks,Textile	Banks,Cement
Banks,Pharma.	Banks,Pharma.	-	Cement,Auto
Banks,Chemicals	-	-	Chemical,Auto
Banks,Auto	-	-	
Banks,Food	Banks,Food	Banks,Food	
Banks,Oil & Gas	Banks,Oil & Gas	Banks,Oil & Gas	
Textile,Pharma.	Textile,Pharma.	Textile,Pharm.	
Textile,O&G	Textile,O&G	Textile,O&G	
Textile,Chemicals	-	-	
Textile,Cement	Textile,Cement	-	
Textile,Auto	Textile,Auto	Textile,Auto	
Textile,Food	Textile,Food	-	
Food,Pharma.	Food,Pharma.	Food,Pharma.	
Food,Oil &Gas	Food,Oil &Gas	Food,Oil &Gas	
Food,Chemicals	Food,Chemicals	Food,Chemicals	
Food,Cement	-	Food,Cement	
Food,Auto	Food,Auto	Food,Auto	
-	-	Cement,Pharma.	
Cement,Chemicals	Cement,Chemicals	Cement,Chemicals	
Cement,Oil &Gas	Cement,Oil &Gas	Cement,Oil &Gas	
Oil&Gas ,Pharma.	Oil&Gas ,Pharma.	Oil&Gas ,Pharma.	
Oil-gas, Chemicals	Oil-gas, Chemicals	Oil-gas, Chemicals	
Oil &Gas , Auto	Oil &Gas , Auto	Oil &Gas , Auto	
Chemicals,Pharma	-	-	
Auto,Pharma.	-	-	

Conclusion

This study examined two pairs of correlations between the stocks of eight sectors of the Pakistan Stock Exchange to estimate the best retention period for investors to benefit from diversification in the Pakistani stock market (PSX). During 2010-2010, 63 publicly traded companies provided daily stock data for the eight sectors. The original aim of this study was to examine the correlation and comovement of company stocks in eight sectors. The companies are listed on the Pakistan Stock Exchange.

The second goal is to determine the holding duration that will allow investors to reap the benefits of diversification. This research is not without limitations; only 10 years, from January 2010 to December 2019, are allotted for this study. The analysis of this study's approaches and techniques is somewhat limited. Additionally, it is restricted to eight of the KSE-100 Index's main sectors on the

Pakistan Stock Exchange (PSX). The analysis of this study is only applicable to the tools and software employed; other tools and software may interpret the findings differently. This study primarily examines the advantages of diversity and the ideal holding duration for investments in the context of the Pakistani stock market. The results collected indicate that investors may profit by making several pair investments. Investors now have a greater selection of the sectors that will provide the most rewards from diversification. It is advised that domestic and foreign investors invest in the best pair of sectors with the specified time periods.

The findings of this study may help Pakistani policymakers decide more wisely among the available investment possibilities. In future research, instead of focusing on sectors, this study might be performed by companies to gain new perspectives. For the purpose of identifying additional investment scenarios, the research time frame may also be increased to 15-20 years. It is possible to use various tools and approaches that can produce various results. Using the right wavelet technique, other combinations, such as the returns of foreign currencies with domestic and international stock markets, should be investigated. The outcomes of wavelet analysis should be empirically tested using real-world or computer-simulated investments in longitudinal study designs.

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